Aspects of topologically gauged BLG/ABJM theories

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Talk at "Iberian Strings 2012", Bilbao
January 31- February 2, 2012

Talk based on:

- *“Three-dimensional N=8 superconformal gravity and its coupling to BLG M2 branes”* with Ulf Gran, `arXiv:0809.4478 [hep-th],` in JHEP
- *"Three dimensional topologically gauged N=6 superconformal ABJM type theories"* with Xiaoyong Chu, `arXiv:0906.1655 [hep-th]`, in JHEP
- *"Higgsing M2 to D2 with gravity: N=6 chiral supergravity from topologically gauged ABJM theory"*, with X. Chu, H. Nastase and C. Pagageorgakis, `arXiv:1012.5969 [hep-th]`, in JHEP
- *work in progress* with U. Gran, J. Greitz, and P. Howe

More references

See also

- *“Superconformal M2-branes and generalized Jordan triple systems”* with Jakob Palmkvist, `arXiv:0807.5134 [hep-th]`, in CQG
- *"Light-cone analysis of ungauged and topologically gauged BLG"*, `arXiv:0811.3388 [hep-th]`, in CQG
- *"D=3, N=8 conformal supergravity and the Dragon window"* with M. Cederwall and U. Gran, `arXiv:1103.4530 [hep-th]`, in JHEP

Some AdS/CFT literature on Neumann boundary conditions and repeated AdS boundaries:

- G. Compere and D. Marolf, `arXiv:0805.1902 [hep-th]`
- S. de Haro, `arXiv:0808.2054 [hep-th]`
- A.J. Amsel and G. Compere, `arXiv:0901.3609 [hep-th]`
- T. Andrade and D. Marolf, `arXiv:1105.6337 [hep-th]`
- T: Andrade and C. F. Uhlemann `arXiv:1111.2553 [hep-th]`

Higher spin:

- M. Vasiliev, `arXiv:0106149 [hep-th]`

Motivation

Three-dimensional conformal field theories are of interest in

- M-theory: M2-branes, AdS_4/CFT_3, etc
- condensed matter: graphene, phase transitions, high Tc, etc
- mathematics: 'monopole operators', etc

Here we will consider "Topologically Gauged BLG/ABJM Theories":

- i.e. matter/Chern-Simons gauge theory with $\mathcal{N} = 8$ (BLG) or $\mathcal{N} = 6$ (ABJM) superconformal symmetry coupled to conformal supergravity
  - their Higgsing (from M2 to D2 branes) to chiral supergravity
  - their (possible) relation to AdS/CFT

Outline:

Systems of N M2 branes with level k

- the $\mathcal{N} = 8$ superconformal theory ($N = 2,\ k = 1, 2$), BLG [Bagger, Lambert] [Gustavsson]
  - the topological gauging of the global symmetries [Gran,BN]
  - work in progress (with U. Gran, J. Greitz, P. Howe)

- the $\mathcal{N} = 6$ theories (any N, k), ABJM [Aharony, Bergman, Jafferis, Maldacena]
  - their topological gauging [Chu, BN]
  - relation to chiral gravity [Chu, BN]
  - the higgsing to D2 branes [Chu, Nastase, BN, Papageorgakis]

Summary and some speculations
("Sequential AdS/CFT", Neumann b.c.)

3-dim $\mathcal{N} = 8$ superconformal field theory : field content

BLG field content:

- 3d scalars X^i_a
  - i: $SO(8)$ R-symmetry vector index
  - a: three-algebra index related to $[T^a, T^b, T^c] = f^{abc}{}_d T^d$ (structure constants f here antisymmetric in a, b, c)
- 3d spinors ψ_a (2-comp Majorana)
  - with a hidden R-symmetry chiral spinor index (also real 8-dim),
- 3d vector gauge potential $\tilde{A}_\mu{}^a{}_b = A_{\mu cd} f^{cda}{}_b$
  - conformal dimensions (deduced from their kinetic terms):
    - -1/2 for X^i_a
    - -1 for ψ_a
    - -1 for A_μ ("kinetic term" = Chern-Simons term) [Schwarz]

3-dim $\mathcal{N} = 8$ superconformal field theory: Lagrangian

The BLG Lagrangian is

$$\begin{aligned}\mathcal{L} &= -\tfrac{1}{2}(D_\mu X^{ia})(D^\mu X^i{}_a) + \tfrac{i}{2}\bar{\Psi}^a\gamma^\mu D_\mu \Psi_a - \tfrac{i}{4}\bar{\Psi}_b\Gamma_{ij}X^i{}_c X^j{}_d \Psi_a f^{abcd} \\ &\quad -V + \tfrac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \tfrac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right) ,\end{aligned}$$

where $D_\mu = \partial_\mu + \tilde{A}_\mu$ and the potential (a "single trace")

$$V = \tfrac{1}{12}(X^i{}_a X^j{}_b X^k{}_c f^{abcd})(X^i{}_e X^j{}_f X^k{}_g f^{efg}{}_d) \,.$$

- can have (quantized) non-trivial level k by rescaling $f^{abc}{}_d$, large k = weak coupling (but on orbifolds $k > 2$ unclear)
- no other free parameters!

BLG transformation rules

The BLG transformation rules for (global) $\mathcal{N} = 8$ supersymmetry are

$$\begin{aligned} \delta X^a_i &= i\epsilon \Gamma_i \Psi^a , \\ \delta \Psi_a &= D_\mu X^i_a \gamma^\mu \Gamma^i \epsilon + \tfrac{1}{6} X^i_b X^j_c X^k_d \Gamma^{ijk} \epsilon f^{bcd}{}_a . \end{aligned}$$

Demanding cancelation on the $(Cov.der.)^2$ terms in $\delta\mathcal{L}$ implies

$$\delta \tilde{A}_\mu{}^a{}_b = i\bar{\epsilon}\gamma_\mu \Gamma^i X^i_c \psi_d f^{cda}{}_b$$

and the fundamental identity [Bagger, Lambert], [Gustavsson]

$$f^{abc}{}_g f^{efg}{}_d = 3 f^{ef[a}{}_g f^{bc]g}{}_d \, ,$$

with the alternative but equivalent form [Gran, BN, Petersson]

$$f^{[abc}{}_g f^{e]fg}{}_d = 0 \, .$$

- one finite dim. realization, $\mathcal{A}_4$, with split $SO(4)$ gauge symmetry (i.e. with levels $(k, -k)$) [Papadopoulos][Gauntlett,Gutowski]

BLG: more properties

- parity: interchanges the two gauge fields for the split $SO(4) = SU(2) \times SU(2)$ with levels $(k, -k)$
- $k = 1, 2$ related to AdS/CFT with *quantum* $U(1)$ sector [Lambert, Papageorgakis],[Bashkirov, Kapustin]
- The field equations for the Chern-Simons gauge field is

$$\tilde{F}_{\mu\nu}{}^{b}{}_{a} + \epsilon_{\mu\nu\rho}(X^i_c \partial^\rho X^i_d + \tfrac{i}{2}\bar{\Psi}_c \gamma^\rho \Psi_d) f^{cdb}{}_{a} = 0$$

i.e. it is not dynamical. In the light-cone gauge one can solve for the entire vector potential!

3-dim $\mathcal{N} = 8$ superconformal gravity

Can the global symmetries of the BLG theory be gauged without adding new degrees of freedom?

- Off-shell field content of 3-dim. $\mathcal{N} = 8$ conformal supergravity :

$$e_\mu{}^\alpha,\ \chi^i_\mu,\ B^{ij}_\mu,\ b_{ijkl},\ \rho_{ijk},\ c_{ijkl},$$

  [Howe,Izquierdo,Papadopoulos,Townsend]

- On-shell Lagrangian = three Chern-Simons-like terms [Gran,BN] (compare $\mathcal{N} = 1$ [Deser,Kay(1983)], [van Nieuwenhuizen], and for any $\mathcal{N}$ [Lindström,Roček])

$$\mathcal{L} = \tfrac{1}{2}\epsilon^{\mu\nu\rho}Tr_\alpha(\tilde{\omega}_\mu\partial_\nu\tilde{\omega}_\rho + \tfrac{2}{3}\tilde{\omega}_\mu\tilde{\omega}_\nu\tilde{\omega}_\rho)$$

$$-ie^{-1}\epsilon^{\alpha\mu\nu}(\tilde{D}_\mu\bar{\chi}_\nu\gamma_\beta\gamma_\alpha\tilde{D}_\rho\chi_\sigma)\epsilon^{\beta\rho\sigma} - \epsilon^{\mu\nu\rho}Tr_i(B_\mu\partial_\nu B_\rho + \frac{2}{3}B_\mu B_\nu B_\rho),$$

- supercovariant spin connection: $\tilde{\omega}_{\mu\alpha\beta}(e_\mu{}^\alpha, \chi^i_\mu)$
- CS terms are of 3rd, 2nd and 1st order in derivatives, respectively

Symmetries of 3-dim $\mathcal{N} = 8$ superconformal gravity

The local symmetries are here

- 3-dim diff's and local $SO(8)$ R-symmetry
- local $\mathcal{N} = 8$ supersymmetry (f^{ν} is the spin 3/2 field strength)

$$\delta e_{\mu}{}^{\alpha} = i\bar{\epsilon}(x)\gamma^{\alpha}\chi_{\mu}, \;\; \delta\chi_{\mu} = \tilde{D}_{\mu}\epsilon(x),$$

$$\delta B^{ij}_{\mu} = -\tfrac{i}{2}\bar{\epsilon}(x)\Gamma^{ij}\gamma_{\nu}\gamma_{\mu}f^{\nu},$$

- local scale invariance

$$\delta_{\Delta}e_{\mu}{}^{\alpha} = -\phi(x)e_{\mu}{}^{\alpha}, \;\; \delta_{\Delta}\chi_{\mu} = -\tfrac{1}{2}\phi(x)\chi_{\mu}, \;\; \delta_{\Delta}B^{ij}_{\mu} = 0,$$

- and local $\mathcal{N} = 8$ superconformal symmetry

$$\delta_S e_{\mu}{}^{\alpha} = 0, \;\; \delta_S\chi_{\mu} = \gamma_{\mu}\eta(x),$$

$$\delta_S B^{ij}_{\mu} = \tfrac{i}{2}\bar{\eta}(x)\Gamma^{ij}\chi_{\mu}.$$

Topologically gauged BLG theory

- This supergravity theory has no propagating degrees of freedom!
  - clear in the light-cone gauge: all non-zero field components (plus ∂_+ on them) can be solved for [BN]
    => "topologically gauged BLG"
- Conformal supergravity coupled to BLG: [Gran,BN]
  - OK to order $(Cov.der.)^3$ and $(Cov.der.)^2$
  - complications at order $(Cov.der.)^1$!
  - the full action not known but see below for ABJM!
- other methods to construct this theory
  - attempts in superspace: "the Dragon window" in 3d [Cederwall, Gran, BN]
    see also [Howe, Izquierdo, Papadopoulos, Townsend]
    and recently [Greitz, Howe],
    [Kuzenko, Lindström, Targaglino-Mazzucchelli]
  - work in progress with Gran, Greitz and Howe.

Topologically gauged BLG theory: details

Supersymmetry to order $(D_\mu)^2$ gives the conformal coupling $-\frac{e}{16}X^2\tilde{R}$: ($f^\mu$ is the dual field strength of the spin 3/2 field χ_μ)

$$L^{top}_{BLG} \;=\; L^{conf}_{grav} + L^{cov}_{BLG}$$

$$+\frac{1}{\sqrt{2}}ie\bar{\chi}_\mu\Gamma^i\gamma^\nu\gamma^\mu\Psi^a\tilde{D}_\nu X^{ia}$$

$$-\frac{i}{4}\epsilon^{\mu\nu\rho}\bar{\chi}_\mu\Gamma^{ij}\chi_\nu(X^i_a\tilde{D}_\rho X^j_a) + \frac{i}{\sqrt{2}}\bar{f}^\mu\Gamma^i\gamma_\mu\Psi_a X^i_a$$

$$-\frac{e}{16}X^2\tilde{R} + \frac{i}{16}X^2\bar{f}^\mu\chi_\mu$$

The terms needed to obtain $\delta L = 0$ at linear and zeroth order in D_μ are lacking (work in progress)

Topologically gauged BLG theory: more details

The extended transformation rules at order $(D_\mu)^2$ in δL are

$$
\begin{aligned}
\delta e_\mu{}^\alpha &= i\sqrt{2}\bar{\epsilon}\gamma^\alpha\chi_\mu\,,\\
\delta\chi_\mu &= \sqrt{2}\tilde{D}_\mu\epsilon,\\
\delta B^{ij}_\mu &= -\tfrac{i}{\sqrt{2}}\bar{\epsilon}\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu - \tfrac{i}{2\sqrt{2}}\bar{\chi}_\mu\Gamma^{k[i}\epsilon X^{j]}_a X^k_a + \tfrac{i}{16\sqrt{2}}\bar{\epsilon}\Gamma^{ij}\chi_\mu X^2\\
&\quad -\tfrac{i}{16}\bar{\Psi}_a\Gamma^k\Gamma^{ij}\gamma_\mu\epsilon X^k_a - \tfrac{i}{2}\bar{\Psi}_a\gamma_\mu\Gamma^{[i}\epsilon X^{j]}_a,\\
\delta X^a_i &= i\epsilon\Gamma_i\Psi^a,\\
\delta\Psi_a &= (\tilde{D}_\mu X^i_a - \tfrac{1}{\sqrt{2}}\bar{\chi}_\mu\Gamma^i\Psi_a)\gamma^\mu\Gamma^i\epsilon + \tfrac{1}{6}X^i_b X^j_c X^k_d\Gamma^{ijk}\epsilon f^{bcd}{}_a,\\
\delta\tilde{A}_\mu{}^a{}_b &= i\bar{\epsilon}\gamma_\mu\Gamma^i X^i_c\Psi_d f^{cda}{}_b - \tfrac{i}{\sqrt{2}}\bar{\chi}_\mu\Gamma^{ij}\epsilon X^i_c X^j_d f^{cda}{}_b.
\end{aligned}
$$

3-dim $\mathcal{N} = 6$ superconformal field theory: ABJM basics

Stacks with more than two M2's require less susy than $\mathcal{N} = 8$ –> ABJM (classical): $\mathcal{N} = 6$, any k and gauge group $U(N) \times U(N)$

- scalar fields now complex Z^A: in bifundamental of the gauge group (A-index a **4** of the R-symmetry $SU(4) \times U(1)$)
- the ABJM version uses no three-algebra f symbols,
- but f can be reinstated [Bagger,Lambert]
- the $k = 1, 2$ ABJM $SU(2) \times SU(2)$ actually has two extra susy's and is then equivalent to BLG
- in general to get $U(N) \times U(N)$ with 8 susy's from ABJM requires 't Hooft monopole operators, but only for $k = 1, 2$ [Klebanov,Klose,Murugan],[Borokhov,Kapustin,Wu],[Gustavsson, Rey]. For new relations between different theories see [Bashkirov, Kapustin].

3-dim $\mathcal{N} = 6$ superconformal field theory: more structure

It is natural to use [BN,Palmkvist]

- $f^{ab}{}_{cd}$ where $[ab]$ and $[cd]$
- $(Z^A_a)^* = Z^a_A$, $(\Psi_{Aa})^* = \Psi^{Aa}$

Then the ABJM theory is (with V on the next slide!)

$$\begin{aligned}\mathcal{L} \quad = \quad & -\tfrac{1}{2}(D_\mu Z^A_a)(D^\mu \bar{Z}^a_A) + \tfrac{i}{2}\bar{\Psi}^{Aa}\gamma^\mu D_\mu \Psi_{Aa} \\ & -\tfrac{i}{2} f^{ab}{}_{cd}\bar{\Psi}^{Ad}\Psi_{Aa} Z^B_b \bar{Z}^c_B + i f^{ab}{}_{cd}\bar{\Psi}_{Aa}\Psi_{Bb} Z^B_c \bar{Z}^d_A \\ & -\tfrac{i}{4}\epsilon_{ABCD} f^{ab}{}_{cd}\bar{\Psi}^{Ac}\Psi^{Bd} Z^C_a Z^D_b - \tfrac{i}{4}\epsilon^{ABCD} f^{ab}{}_{cd}\bar{\Psi}_{Aa}\Psi_{Bb}\bar{Z}^c_C \bar{Z}^d_B \\ & -V + \tfrac{1}{2}\epsilon^{\mu\nu\lambda}(f^{ab}{}_{cd}A_{\mu b}{}^d \partial_\nu A_{\lambda a}{}^c + \tfrac{2}{3} f^{bd}{}_{gc} f^{gf}{}_{ae} A_{\mu b}{}^a A_{\nu d}{}^c A_{\lambda f}{}^e)\,,\end{aligned}$$

and the fundamental identity (solutions classified by Palmkvist)

$$f^{e[a}{}_{dc} f^{b]d}{}_{gh} = f^{ab}{}_{d[g} f^{ed}{}_{h]c}\,.$$

Topologically gauged $\mathcal{N} = 6$ superconformal ABJM

Since ABJM is less rigid than BLG it should be easier to gauge!

- The complete lagrangian has about 25 new terms [Chu, BN]
  - New scalar interaction terms :
    First: Recall the original ABJM potential (single trace in 3-alg.)

$$V = \tfrac{2e}{3}|\Upsilon^{CD}{}_{Bd}|^2,\ \ \Upsilon^{CD}{}_{Bd} = f^{ab}{}_{cd} Z^C_a Z^D_b \bar{Z}^c_B + f^{ab}{}_{cd} \delta^{[C}_B Z^{D]}_a Z^E_b \bar{Z}^c_E\,.$$

    The new terms with one structure constant are (double trace)

$$\tfrac{e}{8} f^{ab}{}_{cd} |Z|^2 Z^C_a Z^D_b \bar{Z}^c_C \bar{Z}^d_D + \tfrac{e}{2} f^{ab}{}_{cd} Z^B_a Z^C_b (Z^D_e \bar{Z}^e_B) \bar{Z}^c_C \bar{Z}^d_D\,.$$

    and without structure constant (triple trace)

$$\tfrac{5e}{12\times 64}(|Z|^2)^3 - \tfrac{e}{32}|Z|^2|Z|^4 + \tfrac{e}{48}|Z|^6\,.$$

  - also new Yukawa-like terms without structure constant
- the transformation rules change: new terms mixing gravity and matter sectors
- all terms in δL checked except a few multi-fermion non-D_μ terms

Higgsing of topologically gauged ABJM

Two observations:

- Higgsing to D2 branes leaves the theory at a chiral point of the Li, Song, Strominger-type [Chu, BN]
- Scaling limits can be taken in different ways [Chu, Nastase, BN, Papageorgakis]

Several steps needed:

- introduce two parameters $\lambda = \frac{2\pi}{k}$ and g_M (via the triple product and the trace)
- expand the theory around a real VEV v: $Z^A = v\delta^{A4} + z^A$
- limits are taken in λ, g_M, v with various combinations kept fixed (including the gauge group)
- identify the six new ordinary supersymmetries

Higgsing of topologically gauged ABJM: the chiral point

The appearance of the chiral point is seen from the scalar/gravitational terms [Chu, BN] (before introducing λ and g_M)

$$L_{higgsed} = L_{CS} - \frac{e}{8}v^2 R - \frac{e}{256}v^6$$

- thus $\frac{v^2}{8} = \frac{1}{\kappa^2}$ and comparison to chiral gravity implies that $\mu = l^{-1} = \kappa^{-2}$, i.e. $\mu l = 1$
- The sign of the Einstein-Hilbert and cosmological terms are as in TMG, i.e., opposite to the signs used by Li, Song and Strominger => negative energy black holes, non-unitarity, etc (see Deser and Franklin)
  - these features are dictated by the sign of the ABJM scalar kinetic terms (via conformal invariance)!
  - introducing more parameters (levels) does not alter this conclusion (see next slide)

Higgsing of topologically gauged ABJM: the scaling limits

After introducing λ and g_M the relevant terms are schematically:

$$
\begin{aligned}
L \;=\; & \frac{1}{g_M^2} L_{CS(\omega)} - |Z|^2 R + \frac{1}{\lambda}(AdA + A^3) - DZD\bar{Z} \\
& -\lambda^2 V^{(1-trace)} - \lambda g_M^2 V^{(2-trace)} - g_M^4 V^{(3-trace)}
\end{aligned}
$$

- in the ABJM formulation (no "f") there are two gauge fields, A^L and A^R with opposite signs of the levels: for higgsing we need to
  - let A^+ and A^- be their sum and difference
  - then the gauge theory CS-term reads: $\frac{1}{\lambda} A^- F^+$
  - the Z kinetic terms gives a term $v^2(A^-)^2$ (with A^+ in D_μ)
  - combining these two last steps gives a kinetic term $\frac{1}{v^2\lambda^2}(F^+)^2$
- thus with $k \to \infty$ we need $v \to \infty$ to keep g_{YM} fixed,
- fixing also the cosm constant requires taking $g_M \to 0$
- the theory is chiral with κ prop. to $\frac{1}{v}$
- no subleading terms after taking the scaling limits

Higgsing of topologically gauged ABJM: the scaling limits

A second version of the scaling limit is obtained multiplying the whole action by g_M^2:

$$L = L_{CS(\omega)} - g_M^2|Z|^2R + \frac{g_M^2}{\lambda}(AdA + A^3) - g_M^2 DZD\bar{Z}$$
$$-\lambda^2 g_M^2 V^{(1-trace)} - \lambda g_M^4 V^{(2-trace)} - g_M^6 V^{(3-trace)}$$

Sending k to ∞ now leads to two fixed but tunable parameters:

- $g_{YM}^2 = \frac{v^2\lambda^2}{g_M^2}$ and $\kappa^2 = \frac{1}{v^2 g_M^2}$

The higgsed lagrangian reads

$$L = L_{CS(\omega)} - (\frac{1}{\kappa^2} + \ldots)R + \frac{1}{g_{YM}^2}(F^+)^2 - \frac{1}{g_{YM}^2}D\tilde{z}D\bar{\tilde{z}}$$
$$-(\frac{1}{g_{YM}^2}(\tilde{z})^4 + subleading) - (\frac{1}{\kappa^2 g_{YM}^2}(\tilde{z})^3 + subleading)$$
$$-(\frac{1}{\kappa^6} + \frac{1}{\kappa^5 g_{YM}}(\tilde{z}) + \frac{1}{\kappa^4 g_{YM}^2}(\tilde{z})^2 + subleading)$$

- "subleading" refers to higher powers of g_{YM}^{-1}

Summary

- Topologically gauged M2 theories exhibit spontaneous symmetry breaking to D2-like theories coupled to chiral gravity
  - Chiral point TMG in topologically gauged ABJM [Chu, BN]
  - Chiral point in BLG: not yet clear!

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- "Sequencial AdS/CFT" ??:

$$AdS_4^{N.b.c.}/TGCFT_3 \rightarrow CPAdS_3/CFT_2$$

- $AdS_4^{N.b.c.}$ = AdS w/ Neumann b.c. ([de Haro][Compere, Marolf])
- $TGCFT_3$ = top. gauged CFT_3
- $CPAdS_3$ = 3d TMG at the Chiral Point

change of foliation in AdS_4 [Andrade, Uhlemann] = 3d higgsing ?

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THANKS FOR YOUR ATTENTION!

Some relevant aspects of AdS4/CFT3: I

Start from the Fefferman-Graham metric [Compere,Marolf] [de Haro]

$$ds^2 = \frac{l^2}{r^2}(dr^2 + (\eta_{ij} + h_{ij}(r,x))dx^i dx^j) \qquad (1)$$

- $h_{ij} = 0$ gives the AdS metric with its boundary at $r = 0$
- $h_{ij}(r,x)$ is the deviation from AdS allowed by Einstein's eq's

$$\bar{h}''_{ij} - \frac{2}{r}\bar{h}'_{ij} + \Box\bar{h}_{ij} = 0, \quad (\bar{h}_{ij} = h^{TT}_{ij}, \ ' = \partial_r) \qquad (2)$$

  - given in even and odd power series in the radial coordinate r

$$\bar{h}_{ij}(r,p) = (1 + ...)h^{(0)}_{ij}(p) + ((pr)^3 + ...)h^{(3)}_{ij}(p) \qquad (3)$$

  - In euclidean signature regularity at $r = \infty$ requires: $h^{(0)}_{ij} = h^{(3)}_{ij}$
- both D and N b.c. are possible (similar to scalars in the BF range)
- at the AdS boundary one may impose

$$\Box^{1/2}h^{(0)}_{ij} = \pm\epsilon_{ikl}\partial_k h^{(0)}_{lj} \qquad (4)$$

Some relevant aspects of AdS4/CFT3:II

This corresponds to adding a grav. Chern-Simons term to the 3d boundary CFT: (below C_{ij} is the Cotton tensor)

- with D b.c.: $h^{(0)}$ is a fixed source and $< T_{ij} >= h^{(3)}_{ij} - C_{ij}(h^{(0)})$
- this defines the usual CFT namely CFT_D
- setting $< T_{ij} >= C_{ij}(\tilde{h}^{(0)})$ defines $\tilde{h}^{(0)}$ as the dual boundary metric

A second CFT, CFT_N, is obtained for Neumann b.c., i.e. $< T_{ij} >= 0$, or $h^{(3)}_{ij} = C_{ij}(h^{(0)})$: we need a dual pair $(\tilde{h}^{(0)}, < \tilde{T}_{ij} >)$

- set $C_{ij}(h^{(0)}) =< \tilde{T}_{ij} >$ and solve for $\tilde{h}^{(0)}$ using the corresponding dual equation above gives a dual pair:
- can be done in momentum space [de Haro] or in light-cone [BN]
  - $h_{++} = -2\partial_-^{-3} T_{2-}$
  - $h_{+2} = -\partial_-^{-3} T_{--}$
  - $\partial_+ h_{++} = 2\partial_-^{-2} T_{2+} - 2\partial_-^{-3}(\partial_2 T_{22})$
  - $\partial_+ h_{+2} = \frac{1}{2}\partial_-^{-2} T_{22} - \partial_-^{-3}(\partial_2 T_{2-})$
  - $\partial_+^2 h_{+2} = -\partial_-^{-1} T_{++} + \partial_-^{-3}(\partial_2^2 T_{22})$

Some relevant aspects of AdS4/CFT3:III

Thus, we have the two dual CFT's of opposite parity both AdS/CFT-dual to the bulk theory:

- $T_{ij} = \tilde{C}_{ij}$ and $\tilde{T}_{ij} = C_{ij}$ in the two CFT's are like electric-magnetic variables
- S-duality between the two CFT's:[de Haro]
  - actually a Legendre transformation: $\tilde{W}(\tilde{h}) = W(h) + V(h, \tilde{h})$
  - with $V(h, \tilde{h}) = CS(h, \tilde{h})$ (= 2'nd variation of the CS functional)

3d boundary theories with supersymmetries

- $\mathcal{N} = 1$ susy: see [de Haro] [Amsel, Compere] [Becker et al]
- $\mathcal{N} = 6$ and $\mathcal{N} = 8$: This talk!